

# A Linearized Model for Flicker and Contrast Thresholds at Various Retinal Illuminances

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# Temporal Contrast Sensitivity Function (TCSF)

Watson, A. B. (1986). Temporal Sensitivity.

Temporal Impulse Function

$$I(t) = A (\Gamma(t, \tau_1, n_1) - a \Gamma(t, \tau_2, n_2))$$

$$\text{TCSF}(f) = S (|(1+i2\pi f\tau_1)^{-n_1} - a (1+i2\pi f\tau_2)^{-n_2}|)$$

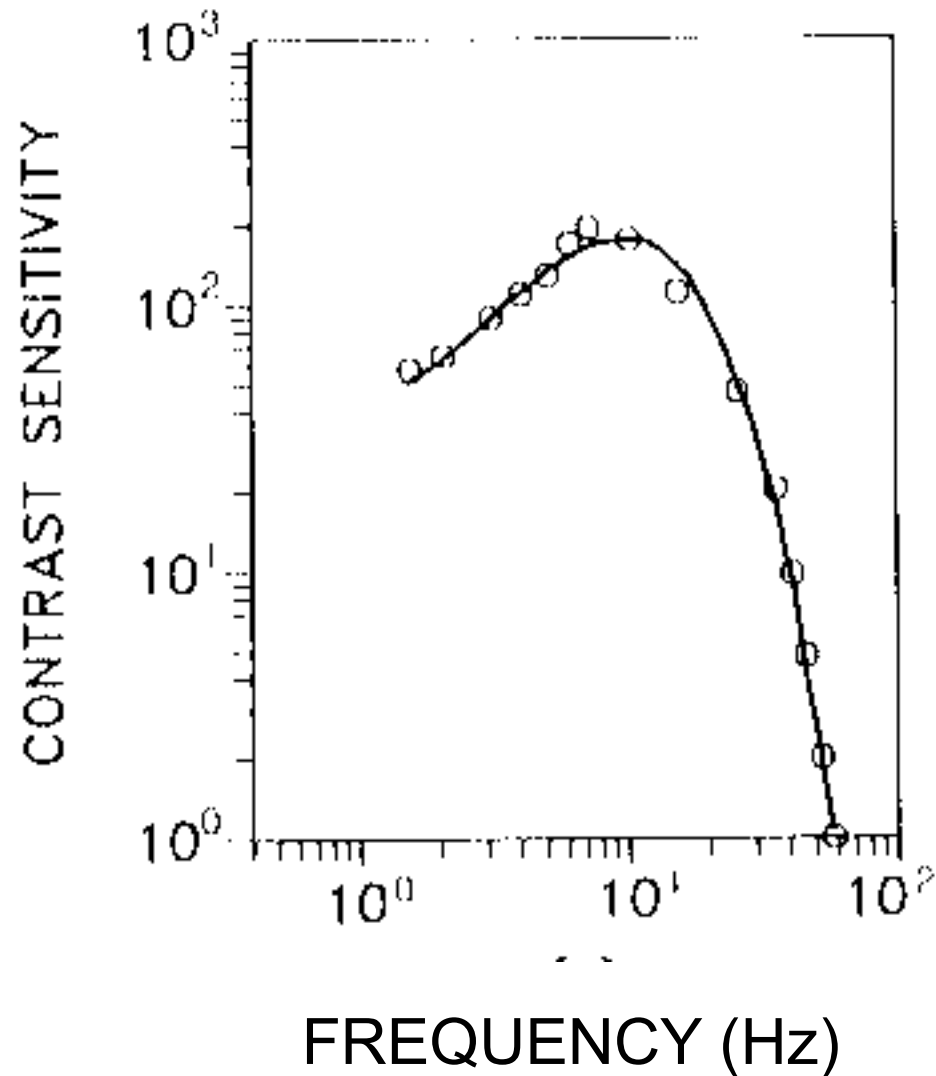
Fit to data of deLange (1958 JOSA)

2 deg field, 1000 Td, observer V

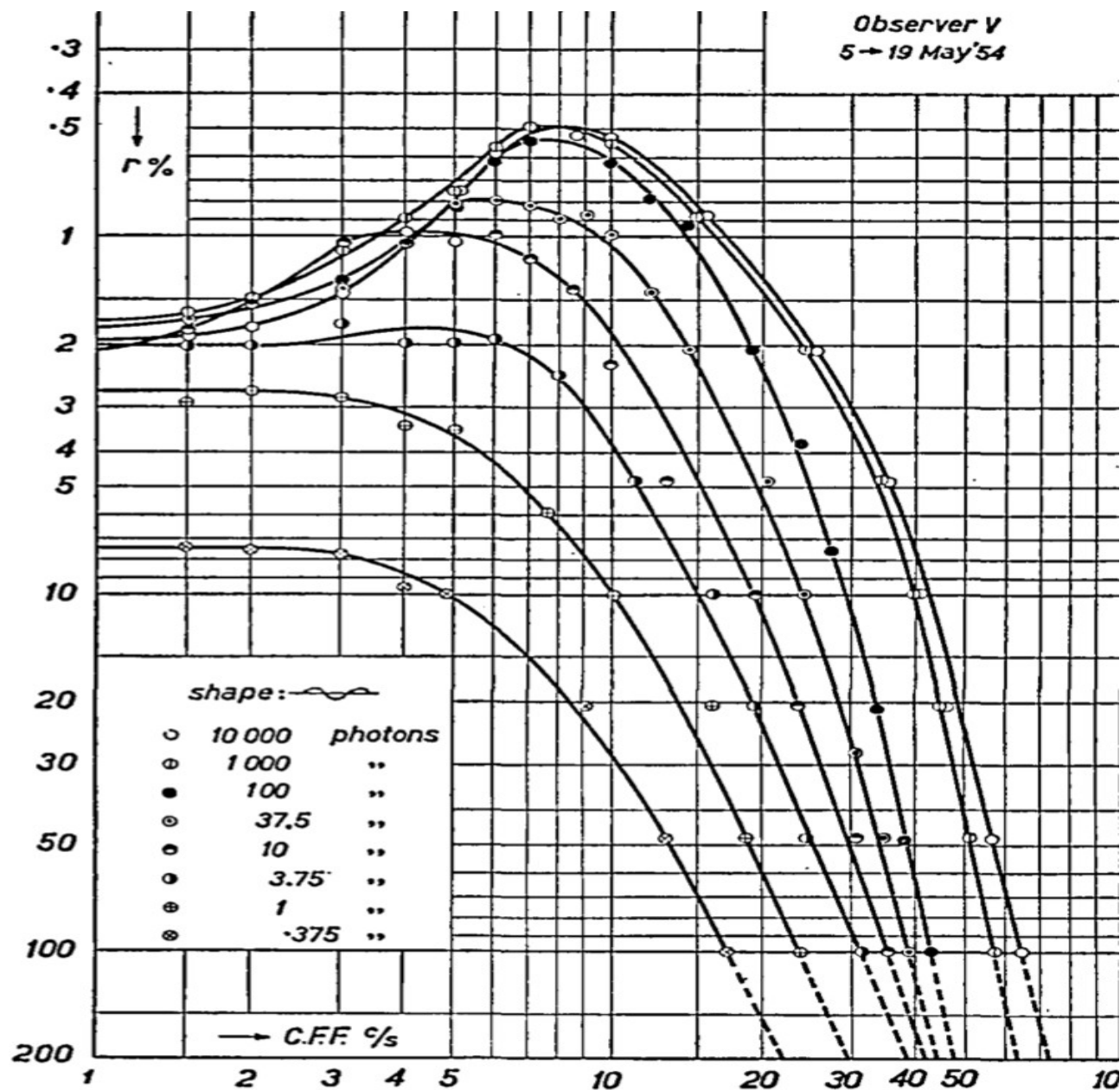
$$S = 269; a = 0.9; \tau_1 = 4.3 \text{ msec}; \tau_2 = 1.33 \tau_1;$$

$$n_1 = 9; n_2 = 10$$

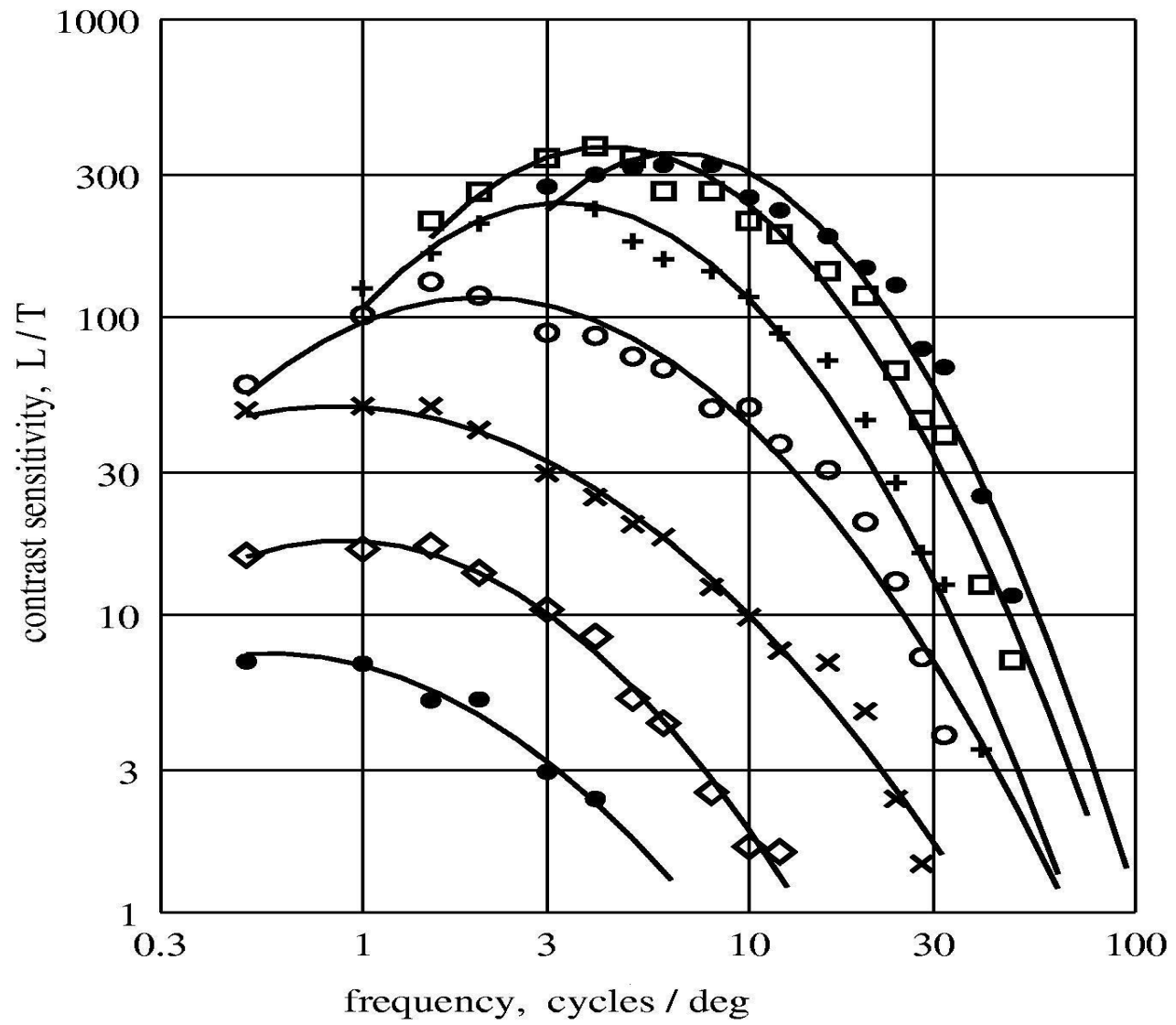
# Temporal Contrast Sensitivity Function (TCSF)



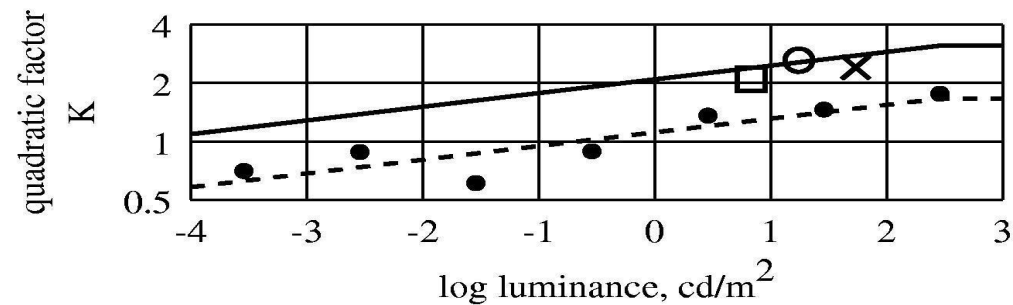
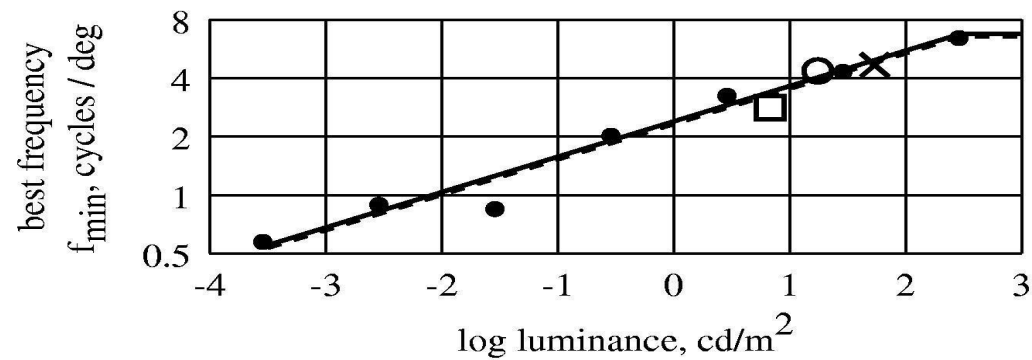
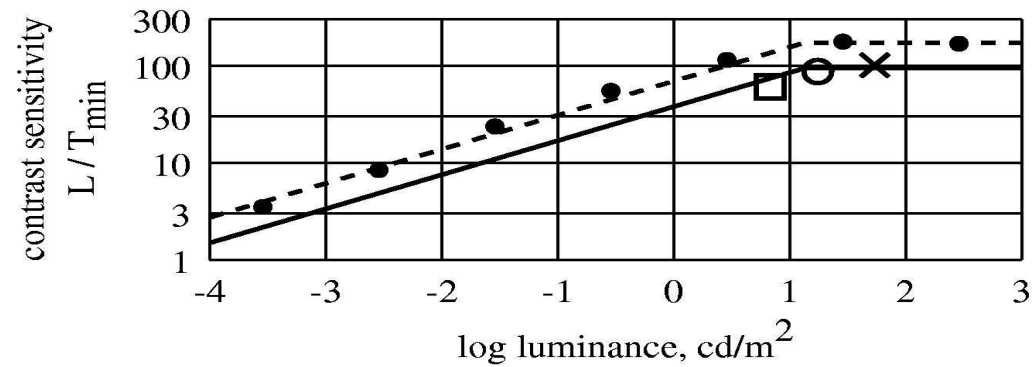
# TCSF(illuminance)



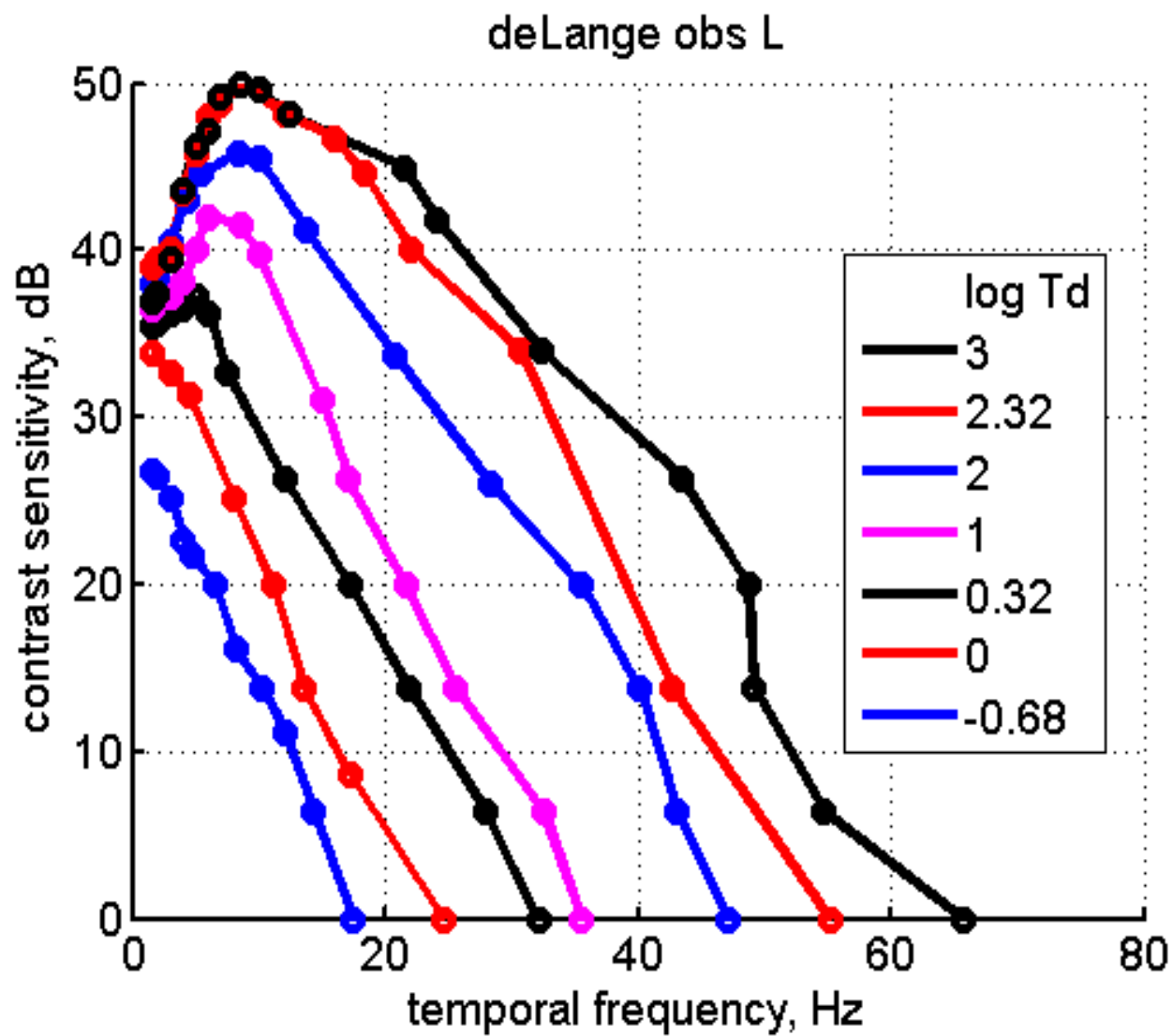
# CSF(illuminance)



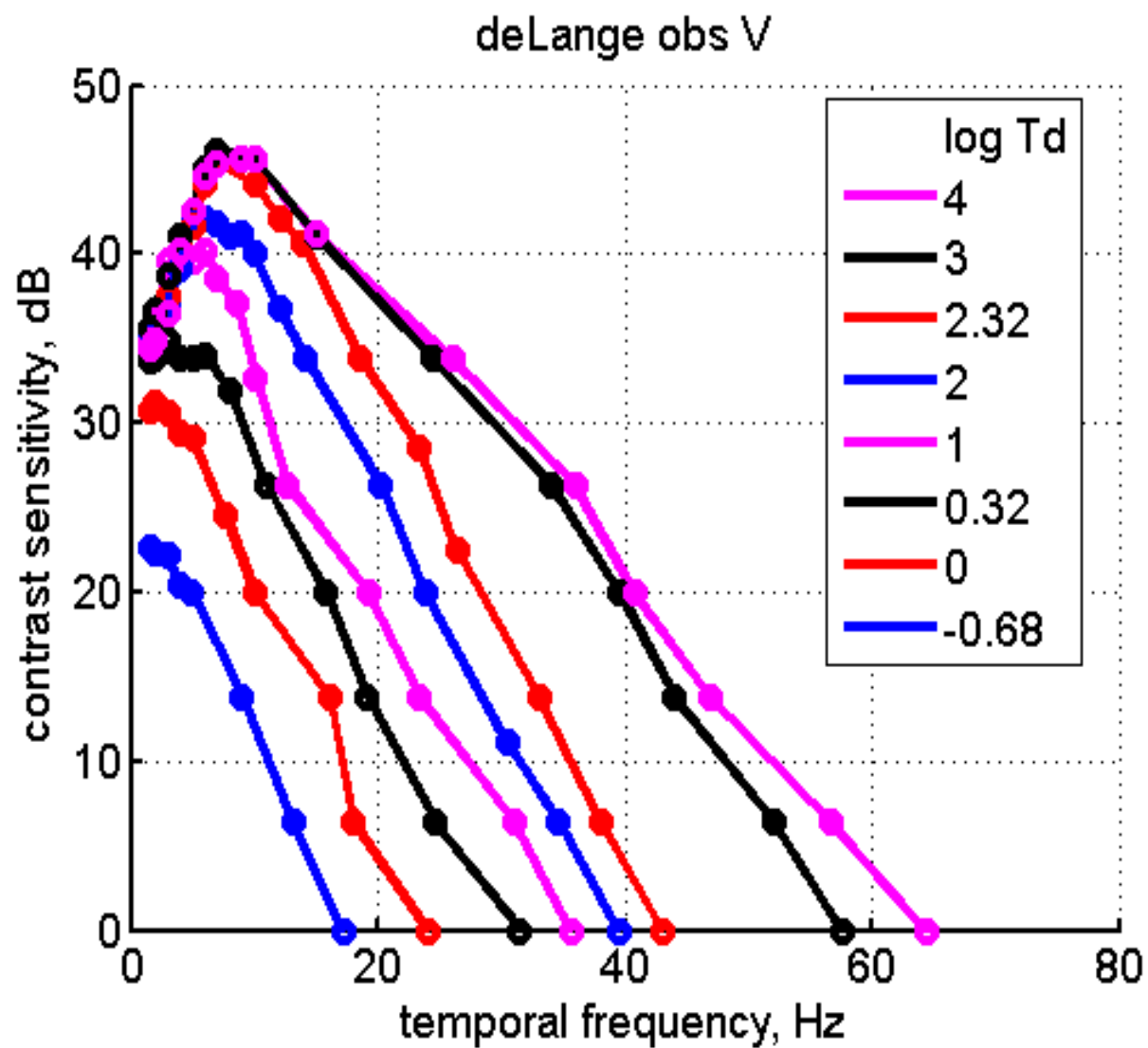
# CSF parameters(illuminance)



# TCSF(illuminance)



# TCSF(illuminance)





# Ferry-Porter Law: Linear relation between CFF and log illuminance

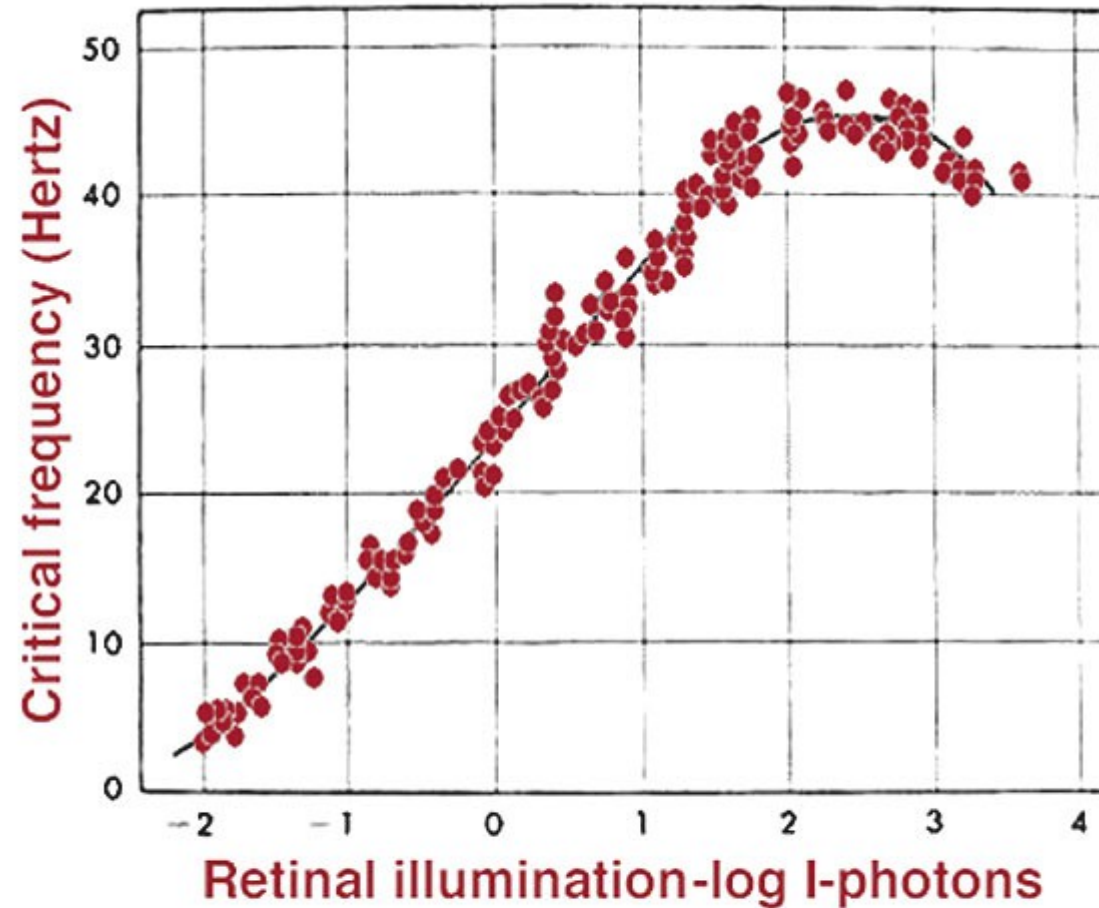


Fig. 7. CFF at the fovea over a range of retinal illuminance (photon = troland) of the test field, showing conformity of the Ferry-Porter Law over four logarithmic units. Hecht and Verrijp's data from Hart Jr, W. M., *The temporal responsiveness of vision*. In: Moses, R. A. and Hart, W. M. (ed) *Adler's Physiology of the eye, Clinical Application*. St. Louis: The C. V. Mosby Company, 1987.

# Bilinear Model

$S$  = log contrast sensitivity

$F$  = temporal frequency in Hz

$I$  = log retinal illuminance in Trolands

$$S = s_0 + s_F F + s_I I + s_{FI} F I$$

or

$$F = f_0 + f_S S + f_I I + f_{SI} S I$$

where  $f_0 = -s_0 / s_F$  ;  $f_S = 1 / s_F$  ;  $f_I = -s_I / s_F$  ; and

$$f_{SI} = 1 / (s_F + s_{FI}) - 1 / s_F ;$$

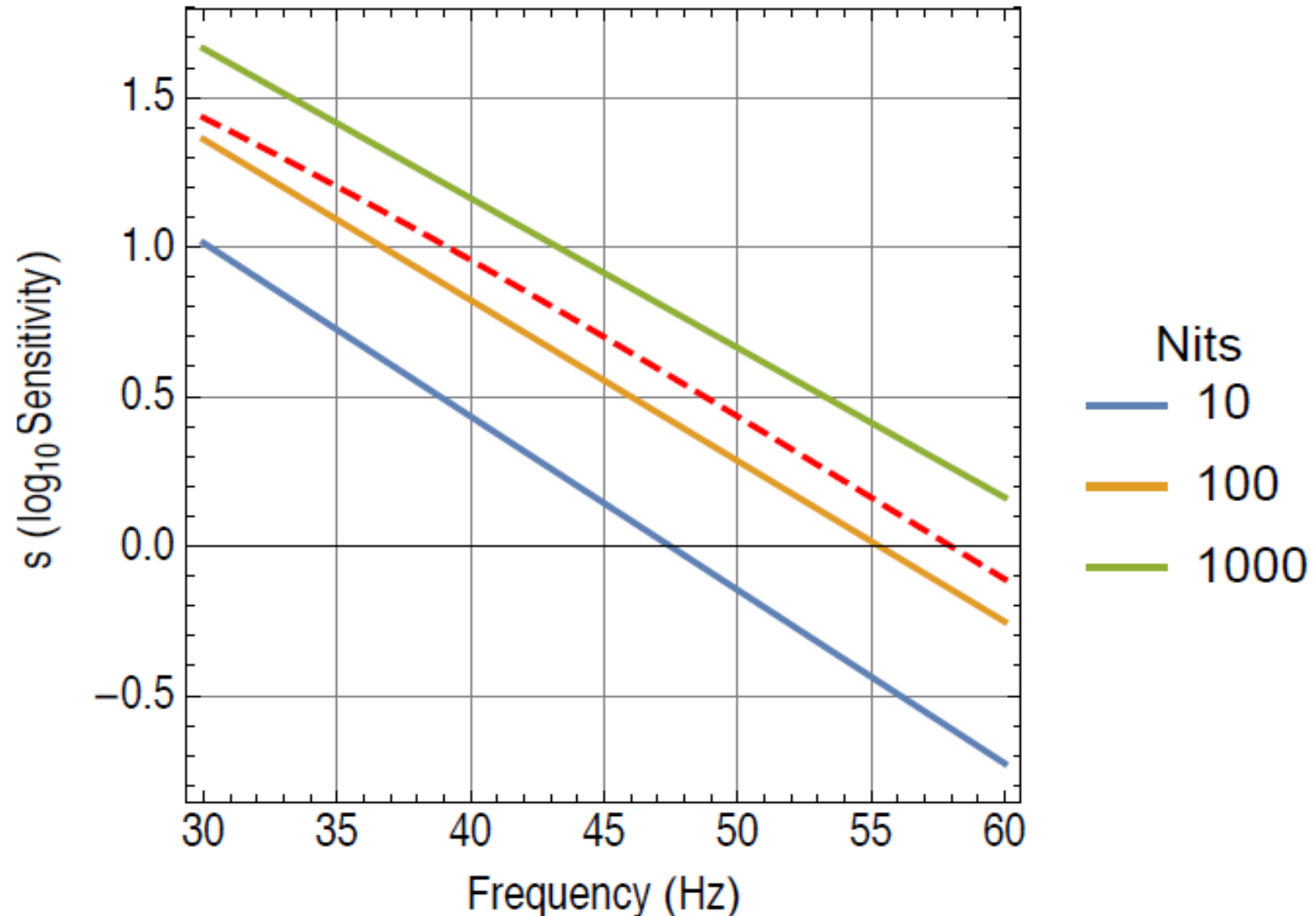
# Bilinear Model Fit

de Lange (1958 JOSA)

obs	n	Smax	RMSE	$f_0$	$f_S$	$f_I$	$f_{SI}$
L	36	1.32	1.5 Hz	23.8	-13.1	12.5	-1.96
V	35	1.46	1.3 Hz	22.9	-13.1	11.1	-4.89

# TCSF( F, luminance )

Pupil size from Watson & Yellott (2012 J Vis)



# Summary

We have provided a bilinear model for the descending-limb portion of the TCSF, based on the approximate linearity of these limbs in log contrast sensitivity and linear temporal frequency and on the assumption that the Ferry-Porter law remains valid for contrast sensitivity values from 0 to 50 dB.

The model fits the flicker threshold data of de Lange (1958 JOSA) with an RMS error of less than 1.5 Hz.

# Acknowledgement

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